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ABSTRACT

Two experiments investigated children's strategies for solving geometric matrices that were correctly or incorrectly completed and that varied in number of elements and number of transformations. Examining the relationship between working memory and item complexity, the first experiment tested 90 boys and girls of 7, 10, and 13 years of age for auditory and visual memory and for performance on Raven's and geometric matrices. Results revealed that 7-year-old subjects were less accurate than either 10- or 13-year-olds; 10-year-olds were slower than 13-year-olds. Developmental differences in strategies included the finding that older children increased latency to solution with increased item complexity, while the youngest group did so only for up to three "feature" matrices. The second experiment, conducted with 30 children, 10 years of age, focused on such matrix characteristics as kind of transformation and nature of falseness. Two questions were raised: Does strategy use differ across addition, expansion, and circular motion transformations? and, Will strategy use vary depending on where the falseness lies in the matrix? Strategic solution seemed to be perceptually driven in addition processes and in circular motion transformations. It was suggested that consideration of what cues are relevant to problem solution appears to be triggered in large part by conceptual/perceptual characteristics of these matrices. Results were considered in terms of the interaction of individual abilities with quantitative and qualitative characteristics of matrix problems. (Author/RH)

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# Children's Solutions for Figural Matrices: Developmental Differences in Strategies and Effects of Matrix Characteristics*

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ABSTRACT

Two experiments investigated children's strategies for solving geometric matrices which were correctly or incorrectly completed and varied in number of elements (1 to 3) and number of transformations (0 to 2). Experiment One tested 90 7-, 10-, and 13-year old boys and girls on auditory and visual memory, the Raven's and geometric matrices. 7-year-olds were less accurate than the older children and 10-year-olds were slower than 13-year-olds. That the older children increased latency to solution with increased item complexity, while the youngest did so only up to three "feature" matrices suggests developmental differences in strategies. Experiment Two was conducted with 30 10-year-olds and focused on such matrix characteristics as kind of transformation and nature of falseness. Strategic solution seemed perceptually driven in addition and circular motion transformations. Results were discussed in terms of the interaction of such individual abilities as working memory capacity and perceptual skill with quantitative and qualitative characteristics of matrix problems.

## Children's Solutions for Figural Matrices: Developmental Differences in Strategies and Effects of Matrix Characteristics

One way in which cognitive psychologists have contributed to understanding the form and content of mental abilities has been to construct cognitive process models of performance on standard psychometric tests (Sternberg, 1981). Hunt (1974) and Jacobs and Vanderventer (1976) have discussed problems of the type presented on the Raven's Standard Progressive Matrices and Stone and Day (1981) found significant increases in 11- and 14-year-olds' and adults' latencies in solution on Raven's-type matrices with increases in the number of elements and number of transformations. Mulholland, Pellegrino, and Glaser (1980) explained the interaction of element and transformation processing by adults on geometric analogies in terms of capacity limitations. They reasoned that processes of pattern comparison-decomposition and transformation analysis-rule generation were essentially serial but that the load on memory of increasing complexity produced a nonlinear pattern of response times.

In an attempt to test the generalizability of a model of analogical reasoning proposed for adults (Sternberg, 1977a, 1977b), Sternberg and Rifkin (1979) presented schematic drawings with separable attributes and with integral attributes to subjects between the ages of 8 and 19. Developmental differences in response latencies and error rates were explained in terms of different patterns of component processes operating in the two kinds of problems. In analogies with perceptually separable attributes change over age appeared due to consistency in use of strategy and component-operation latency and difficulty. However, in analogies with integral attributes developmental differences were attributed to these two processes as well as availability of component operations and strategies for combining multiple executions of the same component operation.

The component processes in Sternberg's (1977) model of analogical reasoning include: (1) encoding each term of the analogy, (2) inferring A-B relationships, (3) mapping A-C relationship, (4) applying these inferences and maps to C to generate an ideal D, (5) (optionally) justifying among alternative answers when none matches the ideal choice, and (6) responding. Pellegrino and Glaser (1982) draw upon Evans' (1968) human performance model in describing the processes of figural analogy solution: (1) pattern comparison and decomposition (encoding or representation), (2) rule generation and matching (relational inference and relational matching), and, optionally, (3) rule discrimination (relative match comparison).

The purpose of the present investigation was to further examine processes and strategies involved in geometric matrix solution, to pursue the relationship between working memory limitations and item complexity, and to study differential demands on processing imposed by such characteristics of matrices as number of elements, kind of transformation and nature of falseness. Experiment 1 hypothesized developmental differences in 7-, 10-, and 13-year-olds' strategies for solving geometric matrices. Tests of auditory and visual memory were administered to see if a positive relationship would be obtained between memory and accuracy and speed on more complex items. Experiment 2 looked, within 10-year-olds, at matrix characteristics and also examined the relationship between individual differences in perceptual abilities and matrix performance. Both Experiment 1 and 2 used Stone and Day's (1981) geometric matrices, which were constructed to be similar to the Raven's problems in the algorithms relating constituent cells; unlike some of the Raven's problems, the matrices did not require disembedding designs. In the first experiment the matrices were projected on a slide projector; in the second they were displayed on a microcomputer. The Raven's Standard Progressive Matrices were administered in both experiments.

## Experiment 1

### Method

Subjects. Subjects were 90 middle-class boys and girls, 30 each from grades 2 (mean age 7;4), and 5 (mean age 10;5), and 8 (mean age 13;4). All of the children attended a Catholic school in Houston, Texas.

### Tests

Measures of working memory. Subjects were administered the Figural Intersection Test (Pascual-Leone, Note 1) and the Backward Digit Span (based on the expanded version of Wechsler, 1974, by Pulos, Note 2) as measures of visual and auditory memory, respectively. The Figural Intersection Test (FIT) is a group administered paper and pencil test consisting of 43 test items, plus six introductory items. Each item consists of two sets of simple geometric shapes, a set on the right called the presentation set and a set on the left called the test set. The presentation set contains a variable number of shapes, physically separated from each other. The test set contains the same shapes as the presentation set, but arranged in an overlapping configuration so that there exists a common area of intersection, which is simultaneously inside all of the shapes. The task is to identify and mark the common area of intersection. The memory demands come from the fact that the presentation and test sets are spaced far enough apart that one must hold the former in mind while searching the latter. Of course, one can look back at the presentation set (hence reducing demands on memory) and individual differences in perceptual abilities would appear relevant in task performance.

The Backward Digit Span (BDS) is individually administered and requires that sequences of numbers presented orally be said in reverse order. This version of the BDS has been expanded according to Pulos (Note 2) to include five trials at classes of difficulty ranging from two to eight digits, similar to the format of the FIT. Subjects were scored on both the FIT and BDS according to the sum of the difficulty

index of each trial passed. For example, passing all five trials with two and three overlapping figures on the FIT yields a score of 10 plus 15, or 25. Passing the same trials on the BDS as well as two trials of four-digit sequences yields a score of 25 plus 8, or 33.

Figural matrices. The Raven's Standard Progressive Matrices, Sets A through E (Raven, 1956) were administered to each grade. Then Stone and Day's (1981) experimental matrices were administered. Each item type in Stone and Day's matrices represent the crossing of one to three elements with zero to two transformations. Geometric forms used were: triangle, rectangle, circle, hexagon, diamond, semicircle, plus sign, and minus sign. Transformations were: no transformation, addition of a form, expansion in size, and circular motion in a plane. For example, in one matrix there is a triangle in each cell of the top row which expands in size. In the second row the same transformation of expansion in size applies to a different element, a circle. In the third row the same transformation applies to an arrow. This matrix of one element and one transformation is "true" because the last cell in the bottom row is completed with the correct element revealing the correct transformation. If a small arrow or a rectangle of any size were in the bottom righthand cell, the matrix would be "false." An example of a true matrix with three elements and no transformation is presented in the top of Figure 1. In the bottom of Figure 1 is a false matrix with two elements and one transformation.

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Insert Figure 1 about here

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## Procedure

Subjects were administered tests in the following order: FIT, BDS, Raven's, and experimental matrices. The experimental matrices were individually administered in two random orders to each subject in two sessions, a training session of twenty

matrices and the test session of 108 matrices (six true and six false matrices representing each of the nine item types). The first two slides served to demonstrate the equipment. When the child hit a "ready" button with his or her non-preferred hand a 1-1/2 second interval timer was triggered. After the interval a slide depicting one of the matrices was released into a carousel projector and projected against a screen. A millisecond timer was activated upon the release of the slide. The child was instructed to "see what's happening in each row" of the puzzle and to hit with his or her preferred hand a blue pad on a response bar if the puzzle was completed correctly and a yellow pad if the puzzle was not. As soon as the response bar was struck the timer was deactivated and the experimenter took note of the latency and correctness of solution.

In addition to the two slides used to demonstrate the equipment there were eighteen training slides representing one true matrix and one false matrix of each item type. If a child made an error on any of these practice items corrective feedback was given by projecting the slide again and asking the child to explain what was happening in each row. Training sessions lasted about twenty minutes and were followed one to three days later by the test session with the 108 experimental matrices. The first ten slides of the experimental set were the same slides shown in the second half of the training session. No feedback was given during the test session. Subjects were reminded to "see what's happening in each row" and to say "right" or "wrong" as they hit the blue or yellow side of the response bar. Test sessions lasted from twenty to fifty minutes per subject with the average being thirty minutes. For no subject did a session have to be discontinued.

Results

Three (grade) x 2 (sex) x 3 (number of elements) x 3 (number of transformations) MANOVAs[1] were performed on number of correct responses and on mean correct and incorrect response time for true and for false matrices separately. Separate

analyses for true and false matrices are appropriate because processing demands and, presumably, strategies for solution are quite different (Mulholland et al., 1980; Pellegrino & Glaser, 1982). For example, in contrast to true matrices, processing of false matrices can be terminated immediately at the point where incorrect information is discovered. Response times which were more than two standard deviations above or below the mean of a subject's response times for each item type were eliminated. Eliminations never exceeded six percent of the responses on an item type for any grade. Mean numbers correct and mean response time on true and false item types are presented in Tables 1 and 2 for each grade.

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Insert Tables 1 and 2 about here

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Analyses of Variance on Accuracy and Response Time

Grade was significant in the MANOVAs both on accuracy and on response time for true matrices, $F(2,84) = 15.14$, $p < .001$ and $F(2,84) = 3.83$, $p < .05$, respectively. However, grade was significant only in the analysis of accuracy for false matrices, $F(2,84) = 7.66$, $p < .001$. Scheffé tests ($\alpha = .05$) indicated that fifth and eighth graders made significantly more accurate responses than second graders. Scheffé tests ($\alpha = .05$) also indicated that fifth graders were significantly slower in responding than the older or younger subjects. There was no significant main effect or interaction due to sex. Significant effects are summarized in Table 3 for true and for false matrices, respectively.

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Insert Table 3 about here

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True matrices. The analysis of number of correct responses yielded the following significant effects: transformation, $F(2,83) = 28.48$, $p < .001$; grade x transformation,

$F(4,166) = 4.03$, $p < .01$, and grade x element, $F(4,166) = 3.51$, $p < .01$. The significant main effect of transformation indicates that accuracy decreased as the number of transformations increased; indeed, the linear component of the transformation effect was significant, $F(1,85) = 57.25$, $p < .001$. However, the grade x transformation and the grade x element interactions reveal that the effect of complexity was greater for the second graders than for the fifth and eighth graders. With increasing complexity (especially in the number of transformations), the accuracy of the second graders decreased more than the accuracy of the older groups, resulting in a larger difference between the youngest and older groups for the most complex items (e.g., 66%-55% correct versus 88%-85% correct) for second versus eighth graders' performance on two transformation matrices. (See 3.96-3.33 and 5.30-5.13 means of Table 1.)

Similar significant effects appeared in the analysis of response times: transformation, $F(2,83) = 59.63$, $p < .001$; element, $F(2,83) = 45.36$, $p < .001$, transformation x element, $F(4,81) = 6.37$, $p < .001$, and grade x transformation, $F(4,166) = 6.43$, $p < .001$. Response times increased as the number of elements increased and as the number of transformations increased. For both transformations and elements there was a significant linear trend, $F(1,85) = 105.62$, $p < .001$, and $F(1,85) = 91.68$, $p < .001$, respectively.

The transformation x element interaction is based on a deviation in the trend towards longer response times for more complex items. For both one- and two-element matrices, response times increased as the number of transformations increased. In contrast, for three element items, response times increased between zero and one transformation but then remained the same for the two transformation items. Perusal of the means in Table 2 indicates that the second graders, whose response times were shorter for two transformation-three-element items than for one-transformation-three-element items, were primarily responsible for this deviation.

The grade x transformation interaction highlights the difference in the effect of more transformations on response times for second graders and for fifth and eighth

graders. The response times of the two older grades increased with more transformations (see Table 2). The response times of second graders increased from zero to one transformation (from 5.14 to 7.95 seconds) and then decreased from one to two transformations (7.95 to 6.15 seconds). The second graders' decrease in response times is particularly interesting in light of their corresponding decrease in accuracy.

False matrices. The analysis of number of correct responses on the false matrices yielded the following significant effects: transformation, $F(2,83) = 76.62$, $p < .001$; element, $F(2,83) = 38.52$, $p < .001$; transformation x element, $F(4,81) = 12.22$, $p < .001$; and grade x transformation x element, $F(8,162) = 3.13$, $p < .01$.

The main effects of transformation and of element indicate that accuracy decreased as the number of transformations and the number of elements increased. The transformation x element interaction, portrayed in the three-way interaction with grade graphed in Figure 2, appears to be based on the atypical performance on the one transformation matrices. That is, there was unusually poor accuracy on the one transformation-three element items and (to a lesser extent) on the one transformation-one element items for all grades.

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Insert Figure 2 about here

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The three-way interaction with grade seems to be based on the greater decrease between one and three elements on the zero transformation matrices for the second graders than for the older groups. (The similarity of the element x transformation pattern for each grade was supported by the significance of these interactions in ANOVAs performed within grade.) In other words, for the younger children more than the older accuracy declined markedly with the addition of a third element in matrices with no transformations.

The following effects were significant in the analysis of response times on false matrices: transformation, $F(2,83) = 42.78$, $p < .001$; element, $F(2,83) = 67.21$, $p < .001$; transformation x element, $F(4,81) = 4.05$, $p < .01$; grade x transformation, $F(4,166) = 3.60$, $p < .01$; and grade x transformation x element, $F(8,162) = 2.57$, $p < .05$.

As was found for true matrices, response times increased as number of elements increased and as number of transformations increased. The transformation x element interaction indicated that the time required to respond to an additional transformation increased as the number of elements increased, and that the time required to respond to an additional element increased as the number of transformations increased.

Figure 3 depicts the three-way interaction of grade, transformation, and element and serves to clarify the grade x transformation interaction as well.

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Insert Figure 3 about here

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Response times for the second graders increased between zero and one transformations, but then remained essentially the same from one to two transformations, especially for the two and three element matrices. Apparently, there was a point in increasing complexity where the second graders stopped processing the additional elements or transformations. Response times for the fifth graders increased with additional transformations and with additional elements. The pattern was similar for the eighth graders at the two transformation level but not at the zero and one transformation levels. The eighth graders responded more quickly to one transformation matrices than to zero transformation matrices except when there were three elements. Such an initially counterintuitive finding might be explained by the older subjects' process termination at the point at which the incorrectness of the transformation was detected. In other words, if a subject's strategy is to first "see what's happening" (i.e., check out the transformation) before identifying and comparing elements, then zero

transformation matrices might take longer to process than single transformation matrices when falseness lies in the execution of the transformation. Experiment 2 was designed to explore such possibilities.

Regression and Correlational Analyses

To understand the relationship between elements and transformations for true and for false matrices within developmental levels a regression was computed for each grade where response time was predicted by free-choice entry of: the linear trend in elements, the quadratic trend in elements, the linear trend in transformations, and the interactions of these trends. For true and false matrices the amount of total variation in response time accounted for by these trends was: 9% and 14% for second grade; 20% and 18% for fifth grade; and 24% and 23% for eighth grade. For fifth and eighth grades predictability clearly lay with the linear trend in elements and the linear trend in transformations ($\alpha = .001$): $F(1,232) = 27.19$ and 47.64 for fifth grade true matrices and 30.83 and 28.26 for fifth grade false matrices; $F(1,232) = 41.79$ and 61.26 for eighth grade true matrices and 54.99 and 43.14 for eighth grade false matrices.

However, in the second grade such strong linearity was evident only in elements for false matrices, $F(1,232) = 39.19$, $p < .001$. In sum, for fifth and eighth graders there was a strong tendency for response time to increase as the number of elements increased from one to three and the number of transformations increased from zero to two. For the second graders this strong tendency was apparent only in elements for false matrices.

The role of visual and auditory memory in figural matrix performance was examined through regression and correlational analyses. For each grade regressions were computed for true and for false matrices where response time summed across the nine item types was predicted by free-choice entry of Figural Intersection Test (FIT) and Backward Digit Span (BDS) scores. The range of FIT and BDS scores was restricted for the second grade but ranges and variances were comparable for fifth

and eighth grades. For true matrices the FIT accounted for 10%, 9%, and 16% of total variation in second, fifth, and eighth grade response times while the BDS accounted for less than 1%, 10%, and 8%, respectively. In the eighth grade the contribution of the FIT was significant, $F(2,27) = 5.73$, $p < .025$. Moreover, regressions computed on eighth graders' response times for each true item type revealed that the contribution of FIT was significant in all but one of the item types with one and two transformations. In brief, the visual measure of working memory--the FIT--predicted variation in response times on complex item types in true matrices for the oldest subjects.

However, for false matrices neither FIT nor BDS contributed significantly to variation in summed response times. Percentage of response time acounted for by the FIT were: 7%, 10%, and 6% for second, fifth, and eighth grades, respectively. For BDS, percentages were: less than 1%, 9%, and 12%, respectively.

Intercorrelations within grade were obtained between scores on the FIT, BDS, and number correct on true and false matrices. The FIT correlated moderately with number correct on false matrices for eighth grade ($r = .50$, $p < .01$) as well as weakly to moderately with number correct on the Raven's Standard Progressive Matrices for eighth and fifth grade ($r = .34$, $p < .05$ and $r = .45$, $p < .01$, respectively). The relationship between performance on the Raven's and performance on the figural matrices was essentially zero, a finding at variance with significant correlations of these measures in Experiment 2 and in Stone and Day's (1981) study.

## Discussion

Experiment 1 had two main purposes: First, to investigate 7-, 10-, and 13-year-olds' strategies for solving geometric matrices; second, to examine the relationship between working memory and item complexity. We will first look at differences in strategy before turning to working memory limitations. The topics are clearly related but less dependent than one might think, given the performance on certain false item types.

These data support the hypothesis of developmental differences in accuracy and latency to solution on figural matrices. Second graders were less accurate than fifth and eighth graders and fifth graders were slower than eighth graders. Second graders' response times did increase on true matrices with one, two, and three elements and no transformations and on one and two element matrices with one transformation. However, as matrix complexity increased beyond this point, response times decreased and errors increased.

The difference between the second graders' and older subjects' performance on figural matrices appears related to strategy use. The fact that second graders' processing time stopped increasing on true matrices involving one transformation with three elements or two transformations suggests a limited strategy. The strategy could account for up to three elements with no transformations or up to two elements with one transformation but no more. Matrices of greater complexity posed demands beyond the scope of the second graders' strategy. Hence, their relatively fast responses but high error rate. In fact, five out of thirty second graders developed a response bias on at least one of the item types containing one or two transformations. Three of these five decided that all matrices with two transformations were wrong. In contrast, no response biases were evident in fifth or eighth graders.

In contrast to the second graders in this study, the fifth and eighth graders' increase in response time with the addition of an element and the addition of a transformation supports for these older subjects a strategy of sequential, exhaustive determination of the elements and of the transformations in each matrix.

Such developmental differences in strategy use generally fit the hypothesis of working memory limitations. Second graders could typically remember no more than three features on the FIT and BDS, the same number they could process on the figural matrices. Fifth graders could typically deal with four to five features on the FIT and BDS and eighth graders with five. With two transformations and three elements

or five features representing the item type with greatest complexity, it is not surprising that fifth graders could achieve eighth graders' level of accuracy with extended processing time (cf., Wagner, Note 3).

But the attempt to find figural matrix performance predicted by individual differences in working memory did not prove very fruitful. The assumption underlying the working memory hypothesis had been that as the number of elements and transformations increased, processing time would have to be devoted to maintaining the accumulated information in working memory, thereby increasing total time for problem solution. As expected, increasing item complexity did not result in simple additive increases in response time. The significant transformation x element interactions for true and false matrices indicated that the time required to process a transformation increased with the addition of an element and that the time required to process an element increased with the addition of a transformation. (In fact, the linear model at each developmental level accounted for less than 25% of total variation in response time.) Measures of auditory and visual memory (i.e., BDS and FIT) had been included in Experiment 1 in an attempt to predict the greater (and nonadditive) demands of more complex items on working memory. FIT scores did significantly predict eighth grade response times on complex true item types but the relationship was positive. That is, as FIT scores increased response times on complex true items increased (with accuracy near ceiling). Such a pattern suggests that with respect to latency to solution the FIT is tapping reflectiveness. However, with respect to accuracy of solution, the significant but moderate relationship for older children between the Raven's and the FIT on the one hand and the figural matrices on the other suggests a common perceptual base. Experiment 2 explores further this possible common perceptual base.

A final point to make is a reminder that speed of solution, at least in the context of this study, was not necessarily reflective of the snowballing of information to be retained in memory. We saw that accuracy was poor for each grade (i.e., 40%

to 46%) on one transformation false matrices and that eighth grade mean latencies appeared reversed on zero and one transformation false matrices with one and two elements. Close examination of these particular matrices suggest that seemingly simple items in terms of numbers of transformations and elements could become cognitively complex in light of subtle perceptual attributes and that self-terminating strategies might be applied which precluded processing of relevant information.

First, an example of the influence of perceptual attributes. In one matrix the transformation was perceptually misleading. The transformation involved was expansion in size. In the bottom row a square with a dash and a triangle in it increased in size from the left cell to the middle. The matrix was falsely completed with a square the same size as the one in the left cell but because of the slightly larger boundaries of the added piece, the square looked larger. Such a perceptual "pull" due to ecological factors creates what Sternberg and Rifkin (1979) call an analogy with "integral attributes," where the attribute of size is integral to the dimensions of the cell (i.e., cannot be nullified). Sternberg and Rifkin (1979) suggest that correct solution of such analogies require more component processes than analogies with perceptually separable attributes. (In Experiment 2 matrices were presented via microcomputer. The perceptual "pull" of this matrix was diminished by use of color and standardization of size increments.)

Second, an example of self-terminating strategies. Accuracy was poor for all subjects on false matrices with one transformation and three elements. In two of the six matrices of this type falseness was due to a change in one of the elements irrelevant to the transformation. In these analogies the attributes were clearly perceptually separable. The fact that error rates were so high for all subjects while response latencies were not suggests a possible short-cutting strategy. That is, subjects might have exhaustively identified elements and specified transformations for only the top row. By the third row, it is possible that only the element undergoing transformation

was scrutinized. In fact, the instructions to "see what's happening" might have encouraged such a focus on the transformation. Consequently, elements not undergoing transformations would be considered less important and a change from a semicircle to a hexagon in the bottom row would not be noticed. Such a pattern is similar to Sternberg's notion of "self-termination" (Sternberg and Nigro, 1980). That is, subjects scan answer options in a top-down fashion until the minimum number of criteria for acceptability are met. Pellegrino and Glaser's (1982) data on false analogy processing also supported a model incorporating self-terminating solution procedures. The data here suggest that such self-termination was a consistent procedure only for the eighth graders where latencies were quicker for false matrices with single transformations than zero transformations when there were one and two elements. In other words, if one's strategy is to first "see what's happening" by examining the transformation, than a matrix where nothing is happening (i.e., zero transformation) requires further consideration and such consideration, presumably of the elements, takes time.

It is this process of consideration, in light of the perceptual and conceptual demands of the matrices interacting with the abilities and proclivities of the subjects, that we turn to in Experiment 2.

**Experiment 2**

Experiment 2 was designed to examine differential demands on problem solution posed by false matrices. Specifically, two questions were raised: Does strategy use differ across addition, expansion, and circular motion transformations? Will strategy use vary depending on where the falseness lies in the matrix--in the transformation and/or in a non-transforming element? Accordingly, in addition to true matrices, only single transformation false matrices with one, two, and three elements were used. Subjects were limited to fifth graders so as to gain maximal power of a within-subjects design. Individual differences were again considered by including the Raven's and the

Figural Intersection Test. The Children's Embedded Figures Test was also included as a measure of field independence-dependence so that a third research question could be raised: Are individuals with good perceptual abilities likely to exhibit different strategic solutions from those with worse perceptual abilities?

Method

Subjects. Thirty fifth grade boys and girls were selected from the same Catholic school (from the same two teachers' classrooms) in Houston, Texas, where Experiment 1 was conducted. Experiment 2 was conducted almost exactly one year later so that the subjects' mean age of 10;4 was virtually identical to that of the fifth graders in the previous study.

Tests. As before, subjects were administered the Figural Intersection Test (Pascual-Leone, Note 1) and the Raven's Standard Progressive Matrices, Sets A through E (Raven, 1956). Instead of Backward Digit Span, the Children's Embedded Figures Test (Witkin et al., 1971) was administered. The CEFT consists of two sets of 12 meaningful designs. The subject is shown a cutout triangle in one set and a cutout house in the other and asked to point to the target in each design.

Stone and Day's (1981) experimental matrices were again administered but this time via an Apple II+ microcomputer and color monitor. Specifically, Stone and Day's 36 one and two transformation true matrices and 18 one transformation false matrices were programmed, with the only change being that stars replaced hexagons so as to improve resolution and color was used. Eighteen additional single transformation false matrices were programmed and were inserted into randomized orders A and B in place of the two transformation false matrices. Zero transformation matrices were omitted altogether as being too easy for fifth graders.

Procedure. Procedures were essentially identical to Experiment 1 with the replacement of BDS with CEFT. Consequently, subjects were administered the FIT, CEFT, the Raven's, and finally, the experimental matrices. As before, subjects were

trained on the experimental matrices before taking the test of 72 matrices. The training consisted of 14 slides, two to demonstrate use of the microcomputer and one true and one false matrix of each item type. (In fact, the six false matrices consisted of two examples--one "new" and one "old"--of the single transformation with one, two, or three elements item types.) A message on the monitor instructed the subject to hit the space bar when ready. One and a half seconds after hitting the space bar, a matrix appeared. The child was instructed to "see what's happening in each row" of the puzzle and to hit with his or her preferred hand a blue button on one game paddle if the puzzle was completed correctly and a yellow button on another game paddle if it was completed incorrectly. The paddles were taped together so they could rest in the child's lap with the buttons facing upward. As soon as a button was pushed the matrix disappeared from the screen and accuracy and response time data were outputted to a Silentype printer out of the child's view. There was a three- to eight-second wait while the computer "drew" the next matrix before it was displayed on the screen. But this wait seemed consistent with the natural pauses subjects exhibited before pushing the "ready" button in Experiment 1.

Results

Analysis of results focused on characteristics of false matrices affecting matrix performance and characteristics of subjects affecting matrix performances. Response time and accuracy on true matrices were examined only in contrast to fifth graders' performance on identical matrices in Experiment 1. Two (group) x 2 (sex) x 3 (number of elements) x 2 (number of transformations) MANOVAs performed on number of correct responses and on response time on correct responses revealed significant differences in accuracy between groups, favoring the fifth graders in Experiment 2, $F(1,56) = 10.18$, $p < .01$. Similarily, MANOVA results on false matrices favored accuracy in Experiment 2, $F(1,56) = 18.27$, $p < .001$. Given that other factors were either controlled (i.e., teachers and school) or comparable (i.e., distributional

characteristics of Figural Intersection Test and Raven's scores), the differences appeared motivated by differing technologies for presenting matrices. Fifth graders were more motivated to solve matrix problems presented via a microcomputer than presented via a conventional slide projector. Heightened motivation led to more rapid, accurate solutions.

Analysis of false matrices in Experiment 2 examined performance with respect to number of elements (1-3), kind of transformation (circular motion, expansion, addition), correctness of transformation, and nature of elements (nested within correctness of transformation). With single element matrices the nesting was incomplete. That is, if a false matrix has only one element and the transformation is correct, then the element undergoing the transformation must be incorrect. There is no other possible source of falseness. The blank sections of the Tables of means for accuracy and response times (see Tables 4 and 5) depict this logically incomplete design.

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Insert Tables 4 & 5 about here

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Consequently, response time and accuracy on one-element matrices were analyzed in 3 (kind of transformations) x 2 (correctness of transformations) x 2 (nature of elements) ~~within subjects~~ multivariate analyses of variance for repeated measures. For two- and three-element matrices, multivariate analyses of variance were conducted which included the additional two-level factor elements. Accordingly, a conservative approach was taken by dividing alpha levels in half (i.e., $p < .025$). Table 6 provides the $F$ values for these single and multiple element analyses of accuracy and response times.

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Insert Table 6 about here

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To check our assumptions from Experiment 1 that response time did not vary as a function of accuracy, we included accuracy as a covariate in analyses of response time. As expected, accuracy of solution proved not to covary significantly with latency to solution for any main effects or their interactions. These fifth graders put time into processing information regardless of the outcome of problem solution. Subsequent analyses considered response time and accuracy data separately.

Accuracy

One-element matrices. From Table 6 we see that in the analysis of accuracy on one-element matrices there were significant main effects of Correctness of Transformation, $F(1,29) = 13.49$, $p < .001$, and Nature of Elements nested within Correctness of Transformation, $F(1,29) = 21.40$, $p < .001$. As we see from the top of Table 4, subjects were more accurate, on the average, on one-element matrices with a correct transformation than an incorrect transformation but the nature of the element within the incorrect transformation significantly affected accuracy. When the transformation was incorrect and the element in the transformation was incorrect subjects made no errors. However, if the transformation was incorrect but the transforming element was correct, subjects made significantly more errors.

Two- and three-element matrices. Referring again to Table 6, we see that in the analysis of accuracy on two- and three-element matrices, there were significant main effects of: Elements, $F(1,29) = 16.97$, $p < .001$; Kinds of Transformations, $F(2,58) = 7.16$, $p < .01$; and Nature of Elements nested within Correctness of Transformation, $F(2,58) = 14.02$, $p < .001$. The following two-way interactions were significant: Element by Correctness of Transformation, $F(1,29) = 26.95$, $p < .001$; Kind of Transformation by Nature of Elements nested within Correctness of Transformation, $F(4,116) = 3.43$, $p < .025$; and Element by Nature of Elements nested within Correctness of Transformation, $F(2,58) = 7.59$, $p < .001$. Finally, there was a significant three-

way interaction: Kind of Transformation by Element by Nature of Elements nested within Correctness of Transformation, $F(4,116) = 3.55$, $p < .01$.

When these effects are examined we see that subjects were more accurate on two-element matrices than three-element matrices and were more accurate on matrices with an expansion transformation than with a circular motion transformation, $t(1,58) = 3.67$, $p < .001$. Moreover, as can be seen from the means of Table 4, subjects were more accurate on matrices with correct transformations when the incorrect element was in the transformation than when it was not. However, when the transformation was incorrect, subjects were more accurate if an element was also incorrect, presumably because of the extra clue to falseness. But closer examination of these means for circular motion, expansion, and addition transformations show us why the nested factor interacted significantly with Kind of Tranformation. Circular motion transformations were difficult, both in correct transformations with incorrect, nontransforming elements and also in incorrect transformations with correct elements (i.e., means of .75 and .70, respectively, in Table 4). Expansion transformations show a rather surprising reversal. Subjects tended to make errors (i.e., mean of .80) on matrices where the element expanded appropriately but another element was incorrect. Yet subjects made relatively few errors (i.e., mean of .93) when the element expanded inappropriately and all other elements were correct.

The means of Table 4 also help explain the Element by Correctness of Transformation interaction. In matrices where the transformation was correct, the addition of a third element resulted in a marked drop in accuracy, while in matrices where the transformation was incorrect, the addition of a third element did not result in a decline in accuracy, presumably again, because an extra cue to falseness was now available. The mean of .69 in Table 4 is also a clue to the interaction of Element with the nested factor and the three means of .60 lend insight into the significant three-way interaction of Kind of Transformation and Element with the nested factor.

The means of .60 represent the thirty subjects' average score on three of the 24 false matrices with multiple elements. A mean of .60 is just above chance and stands in contrast to the means of .80 or better on the other matrices.

Response Time

One-element matrices. In the analysis of response time on one-element false matrices, there were significant main effects of: Kind of Transformation, $F(2,58) = 18.80$, $p < .001$; Correctness of Transformation, $F(1,29) = 23.77$, $p < .001$; and Nature of Elements Nested within Correctness of Transformation, $F(1,29) = 25.85$, $p < .001$. Additionally, there were significant interactions of Kind of Transformation with Correctness of Transformation, $F(2,58) = 8.15$, $p < .001$ and with the nested factor, $F(2,58) = 4.55$, $p < .025$. These results parallel the accuracy results with the additional effect of Kinds of Transformation. Subjects' latency to solution was significantly faster on matrices with correct transformations than on incorrect transformations. But the nature of the element within the incorrect transformation significantly affected response time. Just as we had seen that accuracy was better on matrices with incorrect transformations when the element was also incorrect, we now see that response time was faster as well. The important point here is that the additional cue of the incorrect element did not add to response time. Indeed, it appears that the salience of this additional clue enabled subjects to respond quickly (and accurately), whereas the processing of an incorrect transformation with a correct element seemed to take time and demand that the subject understand the algorithm underlying the transformation.

Significant effects of Kind of Transformation were generally due to longer response times for circular motion and addition transformations. Latencies to solution on matrices with circular motion and addition transformations were significantly slower than those with expansion transformations, $t(1,58) = 5.04$, $p < .001$ and $t(1,58) = 2.69$, $p < .01$, respectively. Only for addition transformations was there a significant increase

in response time for incorrect transformations compared to correct transformations, $F(1,58) = 21.95$, $p < .001$. However, as can clearly be seen by the means of Table 5, both addition and circular motion transformations took a long time to solve when the transformation was incorrect and the element was correct (i.e., 17.20 and 16.96 seconds compared to 7.21 seconds for expansion).

Two- and three-element matrices. As we see from Table 6, the MANOVA on response times for matrices with two and three elements indicated significant main effects of Elements, $F(1,29) = 46.98$, $p < .001$ and Kind of Transformation, $F(2,58) = 19.48$, $p < .001$. Significant two-way interactions were: Element by Kind of Transformation, $F(2,58) = 4.09$, $p < .025$; Kind of Transformation by Correctness of Transformation, $F(2,58) = 12.39$, $p < .001$; Element by Correctness of Transformation, $F(1,29) = 10.57$, $p < .01$; and Element by Nature of Elements nested within Correctness of Transformation, $F(2,58) = 4.00$, $p < .025$. Significant three-way interactions were: Element by Kind of Transformation by Correctness of Transformation, $F(2,58) = 5.96$, $p < .01$; and Element by Kind of Transformation by the nested factor, $F(4,116) = 4.17$, $p < .01$.

Examination of these results reveal that latency to solution differs significantly for each kind of transformation: Circular Motion is significantly slower than Addition, $t(1,58) = 3.30$, $p < .01$, which, in turn, is significantly slower than Expansion, $t(1,58) = 2.94$, $p < .01$. Latencies were significantly slower on matrices with three elements compared to those with two, particularly for Circular motion and Addition transformations. Moreover, we see from the two-way interaction of Element with Correctness of Transformation and the three-way interaction with the nested factor that the disparity between two and three elements was greatest with correct transformations (see means of Table 5), particularly when falseness lay in the nontransforming element, $F(1,58) = 6.71$, $p < .025$.

Finally, by examining the significant relationship between Elements, Kinds of Transformation, and Correctness of Transformation, we find that within three elements response times on correct versus incorrect transformations were significantly different for Circular Motion, $F(1,58) = 5.89$, $p < .025$, and Expansion, $F(1,58) = 26.14$, $p < .001$. In other words, subjects took longer to respond to circular motion matrices when the transformation was incorrect than when it was correct. However, just the opposite happened with expansion matrices: Subjects responded faster when the transformation was incorrect (and Table 4 reminds us that their accuracy was quite high).

Correlations of Subject Characteristics with Matrix Performance

Two sets of correlational analyses were performed. First, scores on the Figural Intersection Test (FIT), Children's Embedded Figures Test (CEFT), and Raven's were intercorrelated with total number correct on true and false matrices. Second, scores on FIT, CEFT, and Raven's were intercorrelated with total number correct on circular motion, expansion, and addition transformations on false matrices. The results of the first analysis revealed that the Raven's correlated significantly with accuracy on true and false matrices ($r = .52$, $p < .001$ and $r = .32$, $p < .05$, respectively) and CEFT ($r = .46$, $p < .01$). Also, CEFT correlated weakly but significantly with accuracy on false matrices, $r = .33$, $p < .05$. (And accuracy on true and false matrices correlated significantly, $r = .39$, $p < .05$.)

A slightly different pattern of results appeared in the second analysis. Accuracy on false matrices with expansion transformations correlated weakly to moderately with FIT ($r = .35$, $p < .05$), Raven's ($r = .36$, $p < .05$), and CEFT ($r = .38$, $p < .05$). Accuracy on false matrices with addition transformations correlated significantly but weakly with Raven's, $r = .35$, $p < .05$, perhaps because Raven's problems often involve addition transformations.

The pattern of overlap in these two sets of results suggests matrix performance depends in part on disembedding abilities represented by the CEFT and Raven's and

more general perceptual abilities represented by the FIT. Although the figural matrices in these studies were designed on algorithmic principles rather than disembedding ones, it is possible that these subjects did, in fact, approach a matrix as rows of patterns from which shapes had to be disembedded. High Raven's and CEFT scorers, then, tended to be accurate on matrix solution. In addition to disembedding ability, a more general perceptual awareness represented by FIT appears to be tapped in expansion transformations. Those who were most accurate on expansion transformations (and highest on FIT) were those who were sensitive to changes in nontransforming elements. In order to locate the common area of overlap in the geometric shapes presented in the FIT, the subject has to identify each shape in the array. Such consistent and exhaustive search assures accurate problem solution in face of a false matrix with a correctly expanding element.

Discussion

The purpose of Experiment 2 was to look at characteristics of false matrices and characteristics of individual subjects affecting strategy use in matrix solution. Subject characteristics focused primarily on perceptual abilities; matrix characteristics centered on the kind of transformation and on the way in which a matrix was falsely completed. Let's first summarize what we have learned about matrix characteristics.

We asked whether strategy use would differ across addition, expansion, and circular motion transformations. The answer is clearly yes. With single element matrices subjects responded faster on expansion transformations than on the other two kinds. With multiple element matrices they not only responded faster to expansions compared to additions or to circular motions, they also responded more accurately to expansions than to circular transformations.

But kind of transformation was not the sole determinant of strategy choice. Kind of transformation also interacted with nature of falseness and number of elements. With single element matrices responses were particularly slow on addition and circular

motion transformations when the transformation alone was incorrect. In other words, there was no other clue to falseness and subjects were simply taking time to execute the transformation. The extra time paid off for accuracy was quite high. In matrices with three elements subjects' responses to circular motion tranformations were slower when the transformation was incorrect than correct, even when one of the other elements provided a double clue to falseness. However, in three-element expansion transformations subjects' responses were slower when the transformation was correct rather than incorrect, just the opposite of circular motion. In both cases subjects appeared to have executed the transformation. But the salience of an incorrect expansion prompted them to immediately decide "False"; whereas an incorrect circular motion led them to look elsewhere for falseness and, perhaps, re-execute the transformation.

What about subjects' accuracy on these transformations? For circular motion matrices errors resulted with incorrect transformations and correct elements and errors resulted with correct transformations and incorrect, nontransforming elements. For expansion matrices errors resulted in the latter case only. That is, when subjects saw an element get Big, Bigger, Biggest, they quickly responded "True," in spite of another element being incorrect. When subjects saw an element Move, Move, and Move, they also (although more slowly because of the greater tracking required) responded "True," in spite of another element being incorrect. Here is evidence, at least in the case of these two transformations, of a self-terminating strategy "Execute transformation; if it executes correctly say 'True' and, [for expansion transformations only] if it executes incorrectly, quickly say 'False'." Clearly it is the perceptual salience of the expanding element which prompted problem solution without consideration of further information. Only those subjects who were able to continue information search beyond the confines of the expansion transformation itself avoided error. Such subjects were the ones who demonstrated exhaustive perceptual searches on the FIT. They were

the one who could track two to ten geometric shapes on the FIT and put a dot in the area of common intersection.

Matrix performance could clearly be improved if children were taught to go beyond the correctness of the transformation (and incorrectness, in the case of expansions) to exhaustively search other sources of information. Such a mandate to "Be thorough: Execute transformations and verify all elements" would resolve, for example, difficulties with two of the three chance-level matrices we discussed in the significant three-way interaction on accuracy. One of thesematrices had a correct circular motion transformation where a diamond changed to a semicircle. The other was a correct expansion transformation where a circle in the quadrant to the left of the expanding element changed to a square. But an important point to make about these two matrices with seemingly subtle perceptual changes is that response times were, on the average, quite slow (means of 16.52 and 15.79, respectively, in Table 5), suggesting some consideration of information beyond the correctness of the transformation. In other words, the process of verifying elements may require quite precise perceptual comparisons.

The other chance-level matrix was an incorrect circular motion transformation with two elements, one inside the other. The elements have rotated one quadrant too far. Of the four matrices with incorrect circular motion transformations and correct elements, two have transformations that rotated one quadrant too far and two have transformations where there is no rotation from the second to third cell.[2] That is, for these latter two matrices, the transforming element in the bottom right hand cell is in the same quadrant as the cell to the left. Subjects can represent the transformation in such matrices as "Moves, Moves, Doesn't Move" and conclude that the solution is false. On the other hand, the representation "Moves, Moves, Moves" was seen to be correct in the case where there were two elements, one inside the other, but not in the case of a single transforming element that rotated one quadrant

too far. Yet number of elements alone was not the problem. Subjects were quite accurate when, in addition to the transforming element rotating one quadrant too far, an incorrect nontransforming element added another clue to falseness. Instead, given the relatively fast response time and low accuracy for this particular matrix, one suspects that the supposedly nontransforming element had, in fact, become integral to the transformation and the representation "Moves, Moves, Moves" had been accepted without verification.

Generally, one can conclude that these fifth graders' understanding of circular motion transformations is not fully developed. Their understanding of addition transformation is well developed because they did not make errors on matrices that had two plus signs, then three, then five. Their relative accuracy on such matrices and their relatively long response times suggest a counting strategy rather than the more perceptual "Some, More, Most" analog to the "Moves, Moves, Moves" and "Big, Bigger, Biggest" representations of the other two transformations.

In particular, we see that once these fifth graders have represented (i.e., encoded) the syntax of the matrix (i.e., the transformation) their strategies for solution tend to be perceptually driven in the case of expansions or conceptually driven in the case of additions and circular motions. As matrices assume these qualitative differences, the inferencing process about truth or falsity will vary. That is, consideration of what cues are relevant to problem solution (i.e., rule generation) appears to be triggered in large part by conceptual/perceptual characteristics of these matrices.

What Experiment 2 tells us, then, is that the nonadditive processing of figural matrices in Experiment 1 is not due to artefacts of particular matrices which strain the validity of particular item types. Even when subjects' working memory capacities are clearly within the boundaries of task demands, as we saw in the case of the fifth and eighth graders in these two studies, certain relevant information will be constrained by the kind of transformation involved. In the case of expansion matrices, the

information beyond the transformation is often ignored, except by those thorough in perceptual search as seen by such measures as the FIT; in the case of circular motion, information regarding the transformation may not be verified. However, the counting strategy invited by addition transformations does seem to encourage such verification. However, verification seems more likely within addition transformations because of the application of a counting strategy.

In sum, Sternberg's (1977) componential model of analogical reasoning and Pellegrino and Glaser's (1982) process model based on Evans' (1968) are right in that they detail steps appropriate to problem solution. But if cognitive psychology wants to understand (and, ultimately, improve) individuals' performance on a psychometric measure such as figural matrices, then we must continue to study the interaction of such individual abilities as working memory capacity and perceptual skill with quantitative and qualitative characteristics of matrix problems.

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## Footnotes

[1]Rather restrictive assumptions required in univariate analysis of variance do not exist in the multivariate approach to analyzing repeated measures data. The reparameterization model utilized in MANOVA carries with it the assumption of equality of variance-covariance matrices across cells. However, the robust nature of the test (particularly with equal Ns) makes it typically a more appropriate test of repeated measures hypotheses (see McCall & Appelbaum, 1973).

[2]One of these four matrices -- one with a transformation where there was no rotation -- was not included in the analyses because it represented a second instance of an incorrect circular motion transformation where the element was correct. We previously pointed out the logical impossibility of single-element false matrices where the transformation was correct but the element was incorrect. In order to ensure that subjects were equally exposed to one, two and three element matrices, two single-element matrices were included where the circular motion transformation was incorrect but the element was correct.

Table 1

Mean Number of Correct Responses on True and False Item Types for Each Grade in Experiment 1

| | Item Type | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Zero Transformation | | | One Transformation | | | Two Transformation | | | |
| Grade | 1E | 2E | 3E | 1E | 2E | 3E | 1E | 2E | 3E | Total |
| True Items: | | | | | | | | | | |
| 2..... | 5.53 | 5.60 | 5.56 | 4.66 | 4.56 | 4.30 | 3.96 | 3.66 | 3.33 | 4.57 |
| 5..... | 5.83 | 5.93 | 5.96 | 5.50 | 5.46 | 5.70 | 5.13 | 5.20 | 5.23 | 5.55 |
| 8..... | 5.73 | 5.95 | 5.90 | 5.53 | 5.50 | 5.30 | 5.30 | 5.20 | 5.13 | 5.50 |
| Total | 5.70 | 5.83 | 5.81 | 5.19 | 5.19 | 5.10 | 4.80 | 4.69 | 4.55 | |
| False Items: | | | | | | | | | | |
| 2..... | 5.23 | 4.90 | 3.96 | 3.70 | 4.73 | 3.33 | 4.23 | 3.83 | 3.93 | 4.20 |
| 5..... | 5.56 | 5.60 | 5.06 | 4.73 | 5.13 | 3.50 | 4.83 | 4.40 | 4.29 | 4.79 |
| 8..... | 5.67 | 5.60 | 5.30 | 4.50 | 4.73 | 3.65 | 4.63 | 4.30 | 4.33 | 4.75 |
| Total | 5.49 | 5.37 | 4.78 | 4.31 | 4.87 | 3.50 | 4.57 | 4.18 | 4.19 | |

Note. - E = element. The range of correct responses on an item type is 0 to 6.

Table 2

Mean Response Time in Seconds on Correct Responses
for True and False Item Types at Each Grade in Experiment 1

| | Item Type | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Zero Transformations | | | One Transformation | | | Two Transformations | | | |
| Grade | 1E | 2E | 3E | 1E | 2E | 3E | 1E | 2E | 3E | Total |
| True Items: | | | | | | | | | | |
| 2.... | 4.02 | 4.63 | 6.78 (5.14) | 7.17 | 8.95 | 7.73 (7.95) | 6.02 | 7.08 | 5.35 (6.15) | 6.41 |
| 5.... | 3.91 | 5.15 | 7.37 (5.48) | 6.11 | 7.67 | 9.87 (7.88) | 8.25 | 10.38 | 11.04 (9.89) | 7.75 |
| 8.... | 2.97 | 3.74 | 5.52 (4.08) | 4.44 | 5.66 | 7.20 (5.77) | 5.90 | 7.23 | 8.03 (7.05) | 5.63 |
| Total | 3.63 | 4.51 | 6.56 | 5.91 | 7.43 | 8.27 | 6.72 | 8.23 | 8.14 | |
| False Items: | | | | | | | | | | |
| 2.... | 4.34 | 5.23 | 7.12 (5.56) | 4.74 | 6.06 | 7.85 (6.22) | 5.48 | 6.34 | 7.90 (6.57) | 6.12 |
| 5.... | 4.75 | 5.61 | 7.40 (5.92) | 6.20 | 6.52 | 9.45 (7.39) | 7.11 | 9.01 | 12.07 (9.40) | 7.57 |
| 8.... | 3.47 | 4.18 | 5.66 (4.44) | 4.29 | 4.98 | 5.38 (4.88) | 5.40 | 6.83 | 8.54 (6.92) | 5.41 |
| Total | 4.19 | 5.01 | 6.71 | 5.08 | 5.85 | 7.56 | 6.00 | 7.39 | 9.50 | |

Note.- E = element. Grade level means for 0, 1, and 2 transformations are in parentheses.

Table 3

Summary Table of F-ratios of MANOVAs Performed on Accuracy and Response Time for True and False Matrices in Experiment 1

| Source of Variation | df | True Matrices | | False Matrices | |
|---|---|---|---|---|---|
| | | Accuracy | Response Time | Accuracy | Response Time |
| Grade | 2,84 | 15.14*** | 3.83* | 7.66*** | NS |
| Transformations | 2,83 | 28.48*** | 59.63*** | 76.62*** | 42.78*** |
| linear | 1,84 | 57.25*** | 105.62*** | 94.35*** | 83.01*** |
| quadratic | 1,84 | NS | 7.32** | 52.74*** | |
| Elements | 2,83 | 1.37 | 45.36*** | 38.52*** | 67.21*** |
| linear | 1,84 | | 91.68*** | 57.13*** | 135.99*** |
| quadratic | 1,84 | | | 32.13*** | 24.46*** |
| G x T | 4,166 | 4.03** | 6.43*** | 1.29 | 3.60** |
| linear | 2,84 | 8.01*** | 13.41*** | | 7.56*** |
| G x E | 4,166 | 3.51** | 2.06 | NS | NS |
| linear | 2,84 | 6.85** | | | |
| T x E | 4,81 | 2.16 | 6.37*** | 12.22*** | 4.05** |
| G x T x E | 8,162 | NS | NS | 3.13** | 2.57* |

* $p < .05$

** $p < .01$

*** $p < .001$

Table 4

Mean Number of Correct Responses in Experiment 2 for One Element False Matrices and Two and Three Element False Matrices with Varying Kinds of Transformations and Nature of Falseness

| Number of Elements | Characteristics of Matrices | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Correct Transformation | | | | | | Incorrect Transformation | | | | | |
| | Incorrect Element in Transformations | | | Incorrect Element not in Transformations | | | Correct Element | | | Incorrect Element | | |
| | Cir | Exp | Addit | Cir | Exp | Addit | Cir | Exp | Addit | Cir | Exp | Addit |
| One Element .....Total | | .99 | | | ——— | | | .76 | | | 1.00 | |
| Two and Three Elements | | | | | | | | | | | | |
| 2 Elements ..... | 1.00 | .97 | .97(.98) | .90 | 1.00 | .96 (.95) | .60 | 1.00 | .83 (.81) | 1.00 | .93 | 1.00 (.98) |
| 3 Elements ..... | .87 | .93 | .97(.92) | .60 | .60 | .87 (.69) | .80 | .87 | .93 (.87) | .93 | 1.00 | 1.00 (.98) |
| Total.. | .93 | .95 | .97 | .75 | .80 | .92 | .70 | .93 | .88 | .97 | .97 | 1.00 |
| | | .95 | | | .82 | | | .84 | | | .98 | |

Note. - Accuracy means range from 0 to 1. The kinds of transformations are: Circular motion, Expansion, and Addition.

Table 5

Mean Response Time in Experiment 2 for One Element False Matrices and Two and Three Element False Matrices with Varying Kinds of Transformations and Nature of Falseness

| Number of Elements | Characteristics of Matrices | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Correct Transformation | | | | | | Incorrect Transformation | | | | | |
| | Incorrect Element in Transformation | | | Incorrect Element not in Transformation | | | Correct Element | | | Incorrect Element | | |
| | Cir | Exp | Addit | Cir | Exp | Addit | Cir | Exp | Addit | Cir | Exp | Addit |
| One Element..... | 10.22 | 7.10 | 6.00 | ——— | | | 17.20 | 7.21 | 16.96 | 7.34 | 4.38 | 7.06 |
| Total..... | | 7.78 | | | | | | 13.79 | | | 6.26 | |
| Two and Three Elements | | | | | | | | | | | | |
| 2 Elements..... | 11.85 | 9.39 | 7.58(9.61) | 9.71 | 7.16 | 9.69(8.85) | 13.46 | 9.77 | 8.81(10.68) | 12.21 | 9.23 | 12.81(11.42) |
| 3 Elements..... | 13.42 | 10.38 | 13.92(12.57) | 15.79 | 16.52 | 15.41(15.90) | 18.10 | 7.66 | 13.33(13.03) | 16.76 | 7.33 | 11.84(11.98) |

Note.- Kinds of transformations are: Circular Motion, Expansion, and Addition.

Table 6

Summary Table of F-ratios for One Element and Two-Three Element MANOVAs on Accuracy and Response Time in Experiment 2

| | | Dependent Variables | | | |
|---|---|---|---|---|---|
| | | Accuracy | | Response Time | |
| Source of Variation | df | 1 Element | 2-3 Elements | 1 Element | 2-3 Elements |
| Elements | 1,29 | -------- | 16.97*** | -------- | 46.98*** |
| Kind of Transformation | 2,58 | .86 | 7.16** | 18.80*** | 19.48*** |
| Ktrn X Elem | 2,58 | -------- | 3.16 | -------- | 4.09* |
| Correctness of Transformation | 1,29 | 13.49*** | .94 | 23.77*** | .00 |
| Ktrn X Corr | 2,58 | 2.13 | 2.15 | 8.15*** | 12.39*** |
| Elem X Corr | 1,29 | -------- | 26.95*** | -------- | 10.57** |
| Ktrn X Elem X Corr | 2,58 | -------- | 3.83 | -------- | 5.96** |
| Nature of Elem w Corr | (1,29)[a] | 21.40*** | 14.02*** | 25.85*** | 1.64 |
| Ktrn X NElem w Corr | (2,58)[b] | 1.37 | 3.43* | 4.55* | 1.16 |
| Elem x NElem w Corr | 2,58 | -------- | 7.59*** | -------- | 4.00* |
| Ktrn X Elem X NElem w Corr | 4,116 | -------- | 3.55** | -------- | 4.17** |

* $p \leq .025$

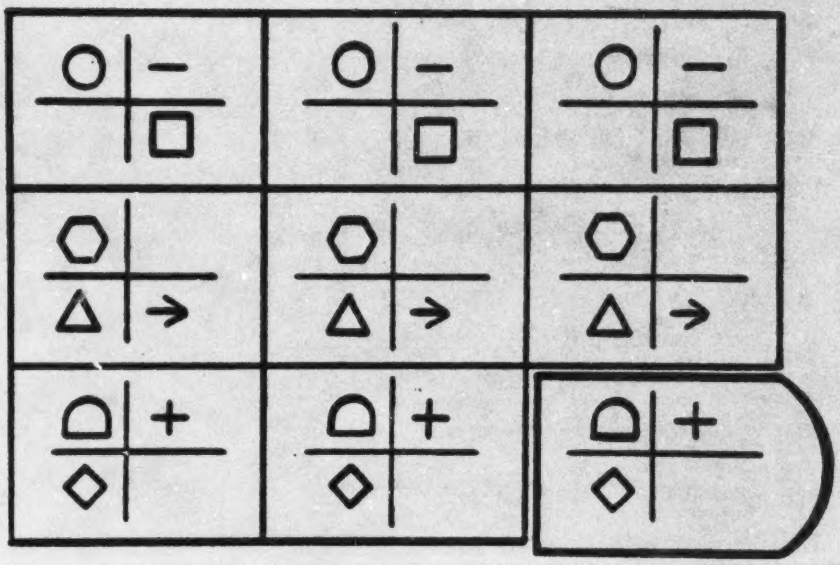
** $p \leq .01$

*** $p \leq .001$

[a] df for 2-3 element analyses = 2,58

[b] df for 2-3 element analyses = 4,116

Fig. 1. - Examples of the experimental matrices; Top: A "true" matrix with three elements and zero transformations; Bottom: A "false" matrix with two elements and two transformations.



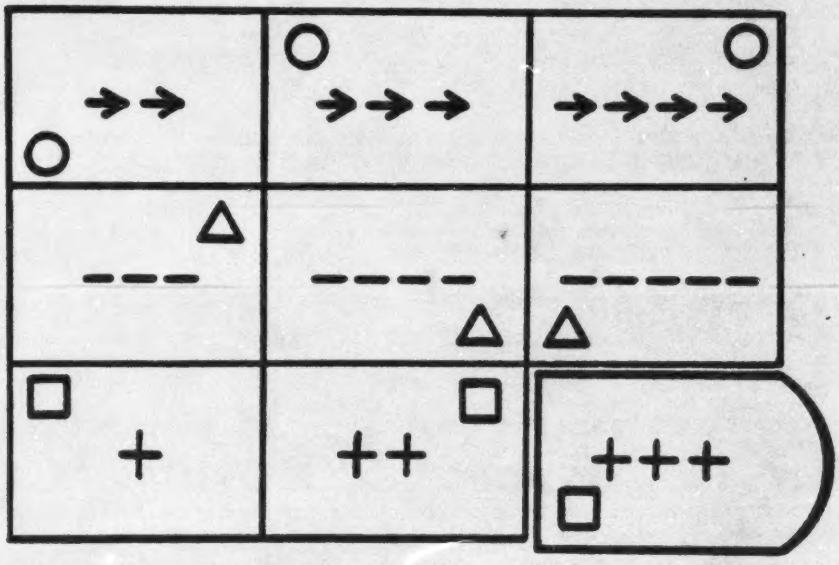


Figure 2

Grade x Transformation x Elements Interaction

for Accuracy on False Matrices

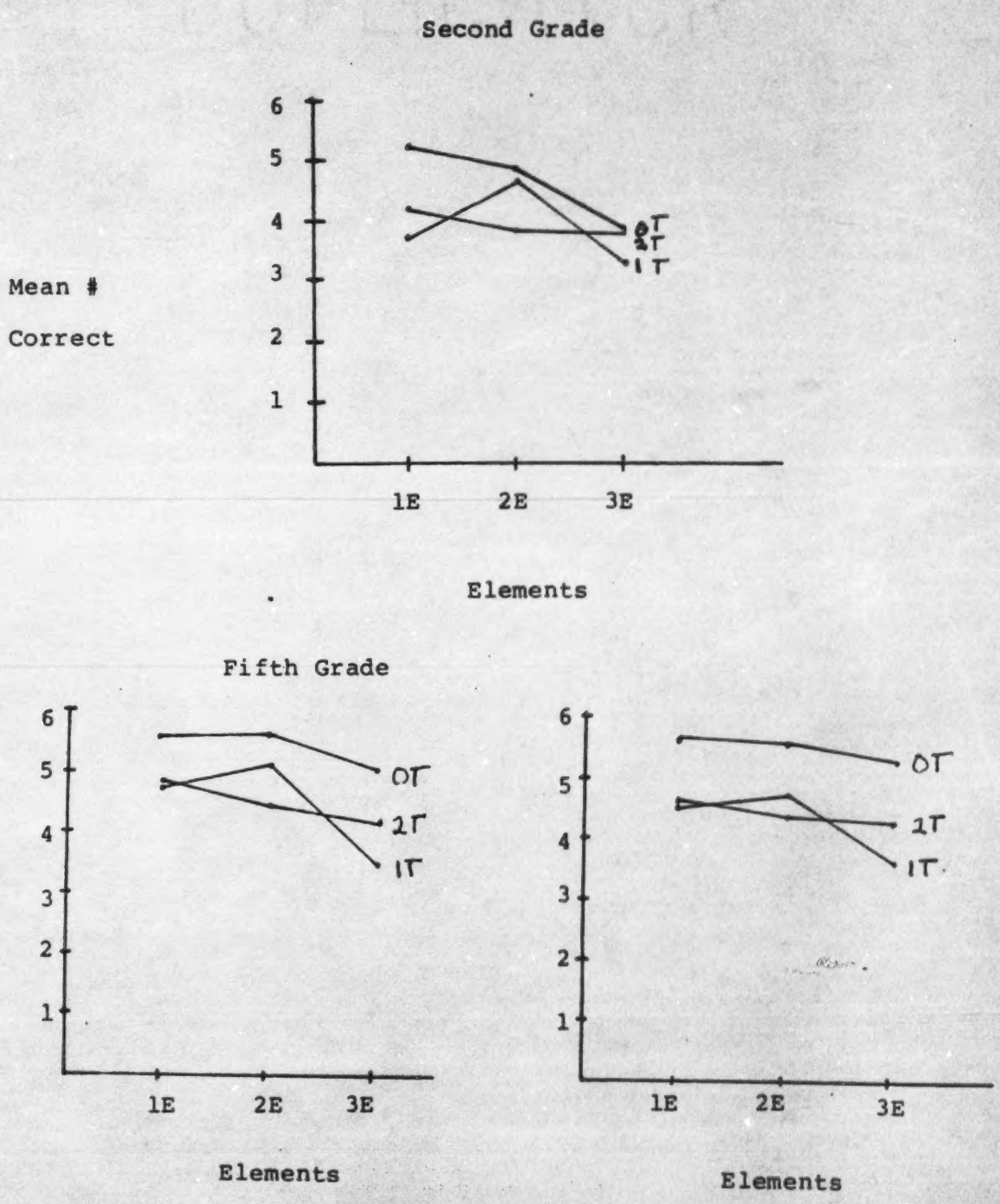


Figure 3
Grade X Transformations X Elements Interaction
For Response Time on False Materials
in Experiment 1